Meta-analysis of Bayesian Analyses

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Meta-Analysis of Bayesian Analyses

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Abstract

Meta-analysis aims to generalize results from multiple related statistical analyses through a combined analysis. While the natural outcome of a Bayesian study is a posterior distribution, traditional Bayesian meta-analyses proceed by combining summary statistics (i.e., point-valued estimates) computed from data. In this paper, we develop a framework for combining posterior distributions from multiple related Bayesian studies into a meta-analysis. Importantly, the method is capable of reusing pre-computed posteriors from computationally costly analyses, without needing the implementation details from each study. Besides providing a consensus across studies, the method enables updating the local posteriors *post-hoc* and therefore refining them by sharing statistical strength between the studies, without rerunning the original analyses. We illustrate the wide applicability of the framework by combining results from likelihood-free Bayesian analyses, which would be difficult to carry out using standard methodology.

Outline

I know it's Saturday. It's a good day for [MBA]!

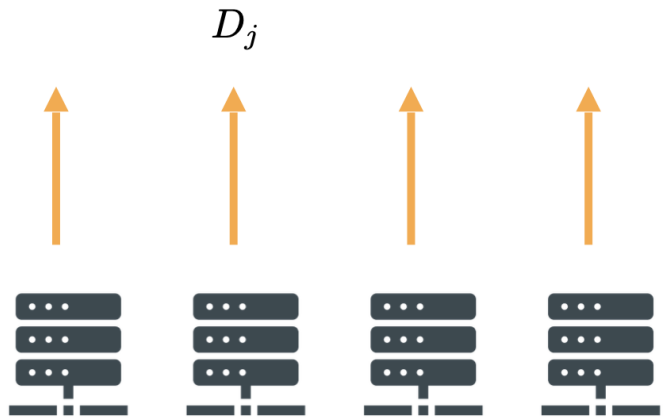
–Inspired by Max Mintz, UPenn.

- What is Meta-analysis?
- Known methods for Meta-analysis
- Meta-analysis of Bayesian Analyses (MBA)
- Example: Using MBA on real data.
- Nice properties of MBA

What is Meta-analysis?

combining results of multiple related studies into a consensus analysis

- Multiple (separate) studies
- Each study uses its own data and modelling assumptions



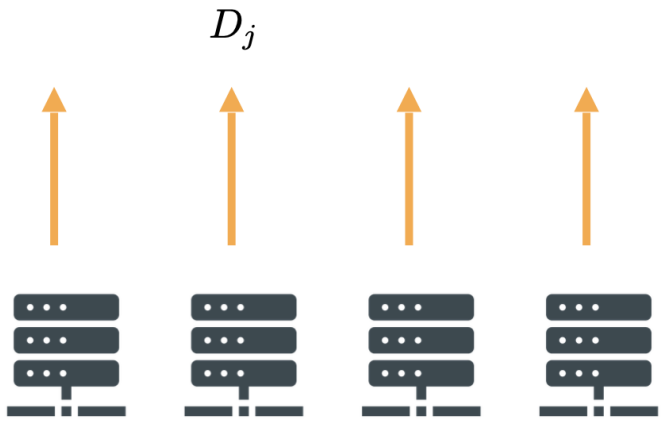
Previously known Meta-analysis methods

- Fixed-effects model.
- Random-effects model.
- Frequentist methods, Bayesian methods.
- In all of these, inputs are point-valued estimates!

Standard Bayesian random effects meta-analysis

Hierarchical model going back to the level of data

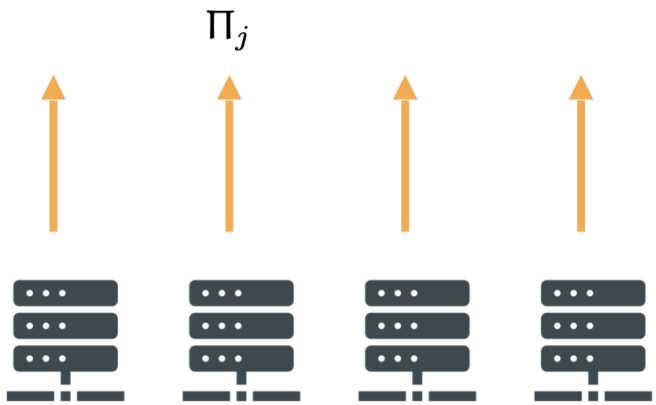
$$\varphi \sim Q$$
$$\theta_j \sim P_\varphi$$
$$D_j \sim F_{\theta_j}$$



$$q(\varphi|D_1,\ldots,D_J) \propto \prod_{j=1}^{J}\left[\int f_j(D_j|\theta_j)p(\theta_j|\varphi)d\theta_j\right] q(\varphi)$$

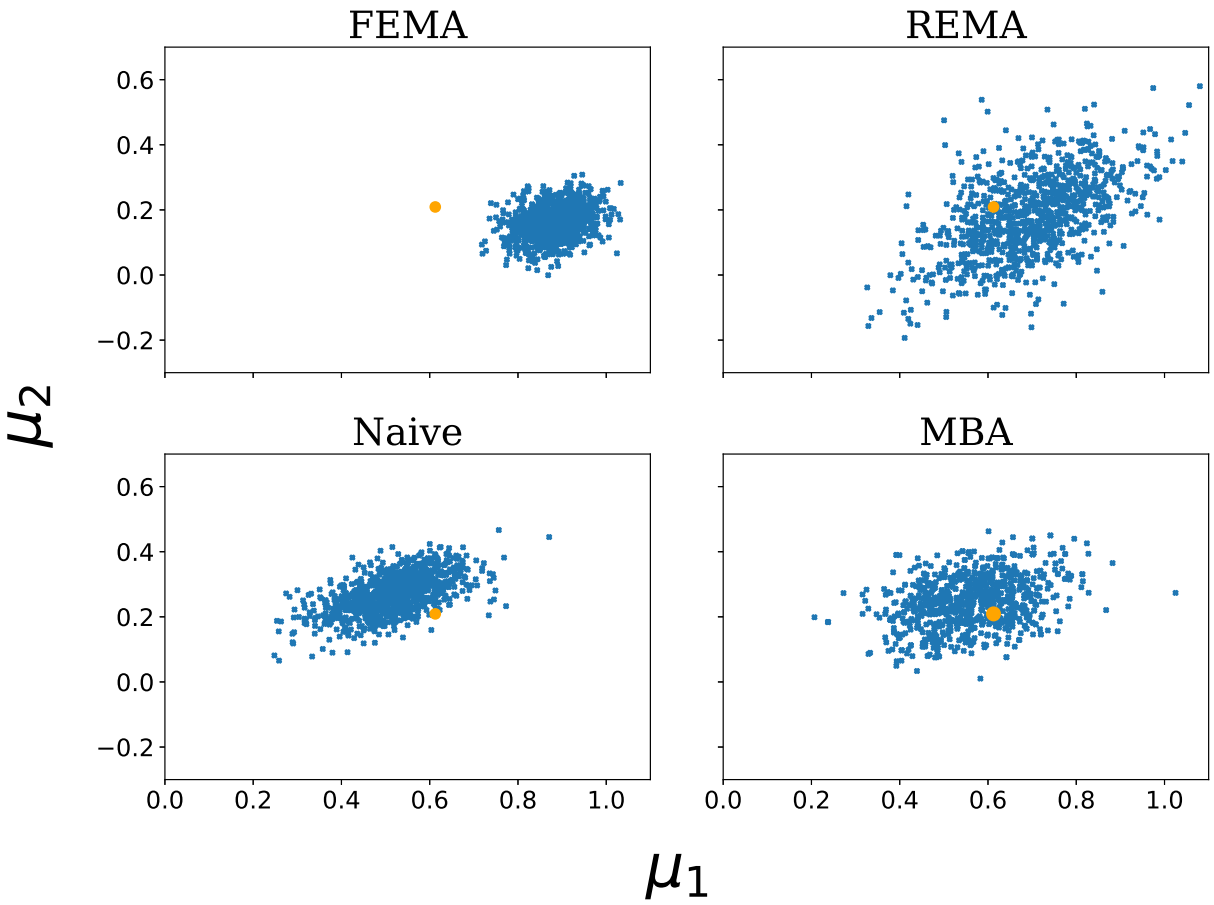
Meta-analysis of Bayesian Analyses

What if we are given a set of posterior distributions, and cannot go back to data?

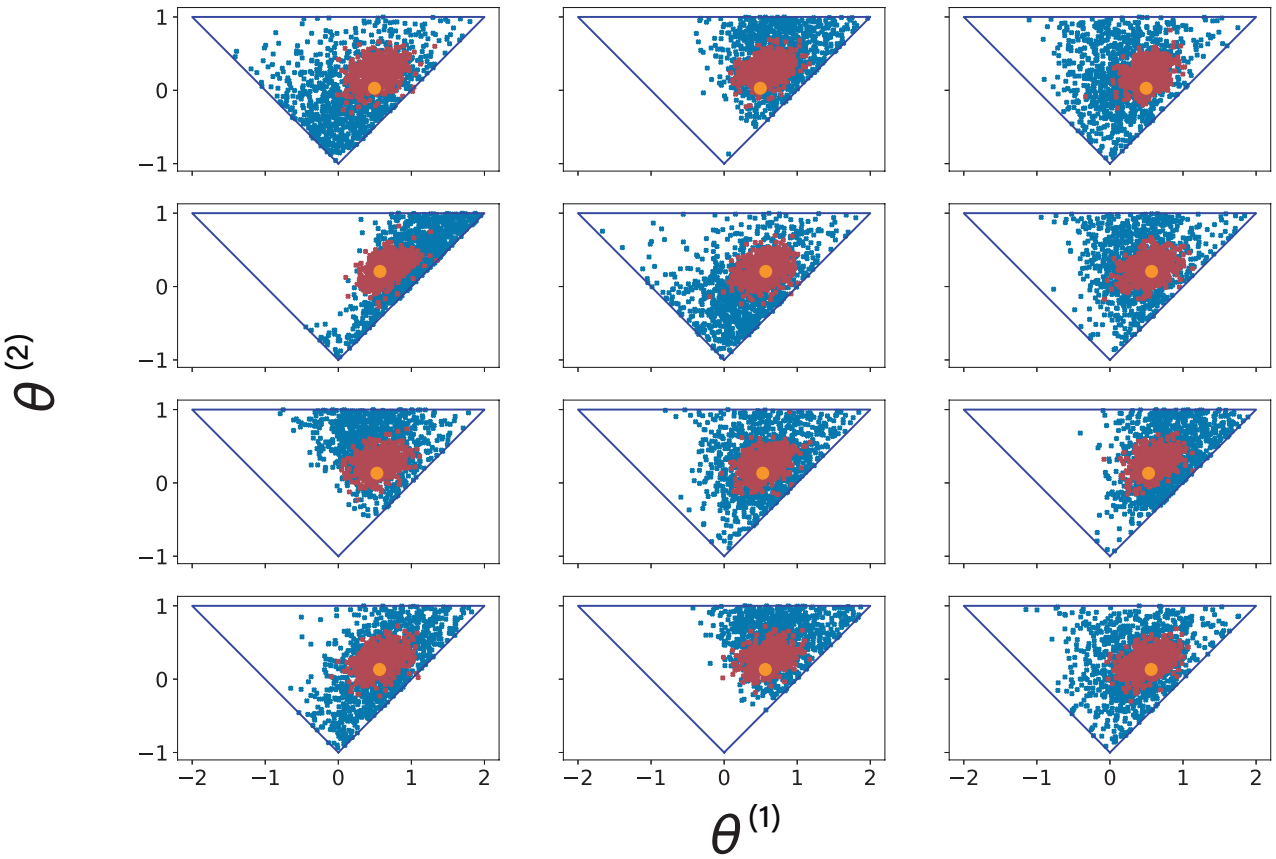


$$q^*(\varphi|\pi_1,\ldots,\pi_J) \propto \prod_{j=1}^{J}\left[\int p(\theta_j|\varphi)\pi_j(\theta_j)d\theta_j\right] q(\varphi)$$

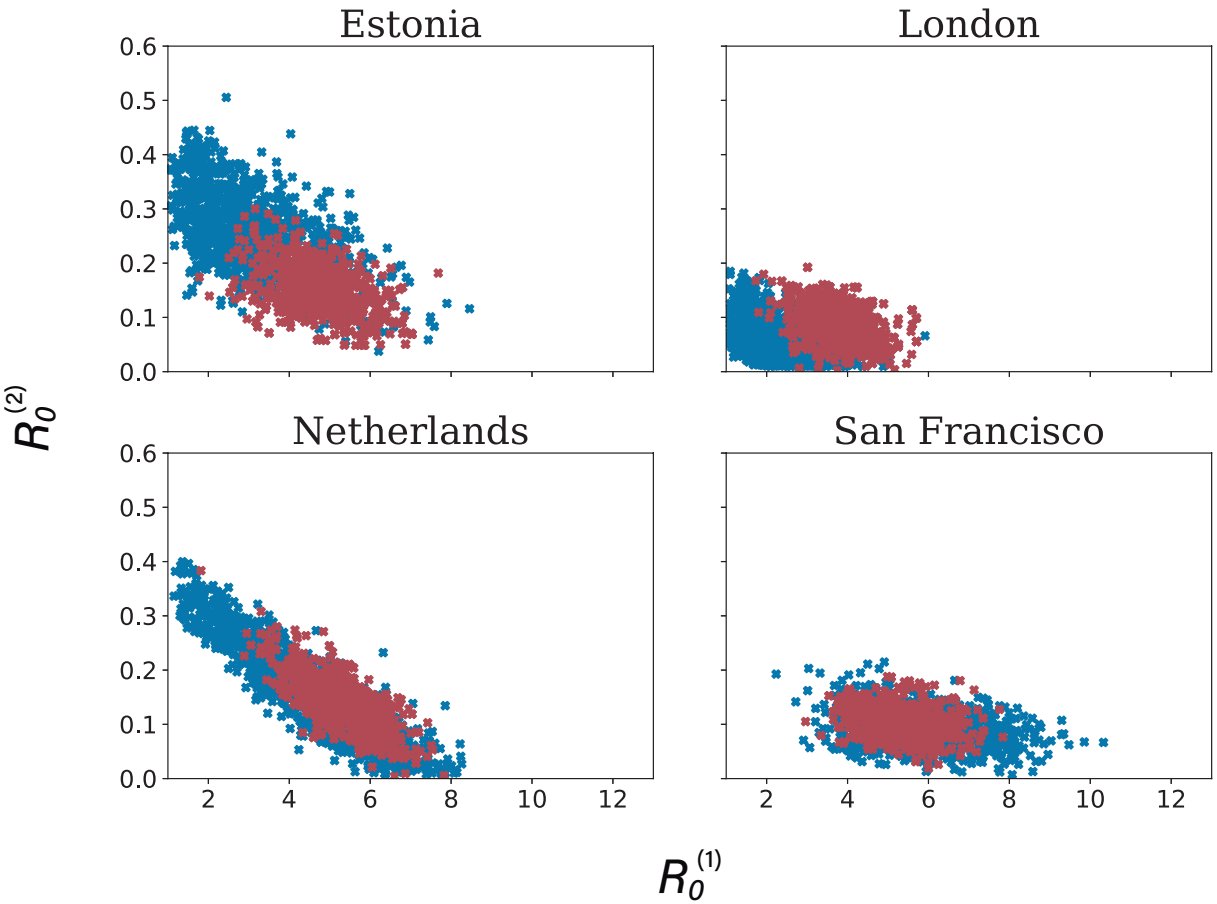
Synthetic example: Overall effect



Synthetic example: Local effects



Real-data example: Tuberculosis outbrake



MBA selling points

$$q^*(\varphi|\pi_1,\ldots,\pi_J) \propto \prod_{j=1}^{J}\left[\int p(\theta_j|\varphi)\pi_j(\theta_j)d\theta_j\right] q(\varphi)$$

- Theory shows:
  - order-invariance in successive updates ✓
  - concentration in the limit of infinite observations ✓
- Experiments show:
  - can reuse pre-computed posteriors from multiple analyses ✓
  - can share strength across studies to refine beliefs on local effects ✓
  - implementation cost is reasonable ✓
- Noteworthy: MBA is a form of 'distributional' meta-analysis.

Last but not least

Many thanks!